APPENDIX D

PERMANENT REGIME WAVE SOLUTION

A nonsteady shock will approach a steady shock in time.[57] A solution for the steady shock is given here based on the constitutive relation of the Horie-Duvall model.[20]

D.1. General Solution

The differential flow equations in Eulerian coordinates are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \ , \tag{D.1}$$

$$\rho \frac{du}{dt} = - \frac{\partial P}{\partial x} \ , \tag{D.2}$$

$$\frac{dE}{dt} = -P \frac{dV}{dt} \ , \tag{D.3}$$

$$\rho \equiv \frac{1}{V} \ .$$

The total time derivative is $(d/dt) = (\partial/\partial t) + u(\partial/\partial x)$. The steady state condition is that $\partial/\partial t = 0$ which, applied to the flow equations, results in

$$\rho u = m = \text{constant} \ , \tag{D.4}$$

$$P + mu = \text{constant} \ , \tag{D.5}$$

$$E - \frac{1}{2}\left(\frac{P}{m}\right)^2 = \text{constant} \tag{D.6}$$

where P represents uniaxial stress. Combining Eqs. (D.4) and (D.5) results in a useful equation which defines the compression path as a straight line in the P-V plane connecting the initial and final P-V states. The equation is

$$P - P^* = -m^2(V-V^*) \ . \tag{D.7}$$

Another useful relation is obtained from Eqs. (D.1) and (D.2):

$$\frac{dV}{dx} = -\frac{1}{m^2}\frac{dP}{dx} \ . \tag{D.8}$$

In the mixed phase region the extensive parameters V and E are defined as functions of P, T, and f such that:

$$V(P,T,f) = V_1(P,T) + f[V_2(P,T) - V_1(P,T)] \ , \tag{D.9}$$

$$E(P,T,f) = E_1(P,T) + f[E_2(P,T) - E_1(P,T)] \ . \tag{D.10}$$

Since P, T, and f are implicit functions of x, Eq. (D.9) and (D.10) result in

$$\frac{dV}{dx} = \left(\frac{\partial V}{\partial P}\right)_{T,f}\frac{dP}{dx} + \left(\frac{\partial V}{\partial T}\right)_{P,f}\frac{dT}{dx} + \left(\frac{\partial V}{\partial f}\right)_{T,P}\frac{df}{dx} \ , \tag{D.11}$$

$$\frac{dE}{dx} = \left(\frac{\partial E}{\partial P}\right)_{T,f}\frac{dP}{dx} + \left(\frac{\partial E}{\partial T}\right)_{P,f}\frac{dT}{dx} + \left(\frac{\partial E}{\partial f}\right)_{T,P}\frac{df}{dx} \ . \tag{D.12}$$

Using Eqs. (D.8), (D.9), and (D.10) and the thermodynamic identities for specific heat C_P, compressibility K_T, and thermal expansion β, then Eqs. (D.11) and (D.12) become: